

Exercise 5

Find dy/dx by implicit differentiation.

$$x^2 - 4xy + y^2 = 4$$

Solution

Differentiate both sides with respect to x .

$$\begin{aligned}\frac{d}{dx}(4) &= \frac{d}{dx}(x^2 - 4xy + y^2) \\ 0 &= \frac{d}{dx}(x^2) - 4\frac{d}{dx}(xy) + \frac{d}{dx}(y^2) \\ &= (2x) - 4\left\{\left[\frac{d}{dx}(x)\right]y + x\left[\frac{d}{dx}(y)\right]\right\} + \left[2y \cdot \frac{d}{dx}(y)\right] \\ &= 2x - 4[(1)y + x(y')] + 2y(y') \\ &= 2x - 4y - 4xy' + 2yy'\end{aligned}$$

Divide both sides by 2.

$$0 = x - 2y - 2xy' + yy'$$

Solve for y' .

$$y' = \frac{2y - x}{y - 2x}$$